Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) 9740/02

Paper 2 **26 September 2016**

3 hours

Additional Materials: Answer Paper
Graph paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Score												
Max Score	**8**	**9**	**11**	**12**	**3**	**6**	**7**	**10**	**10**	**12**	**12**	**100**

Section A: Pure Mathematics [40 marks]

1 The function f is defined by

$$\mathrm{f} : x \mapsto \pi \sin\left(\tfrac{1}{2}x\right), \quad x \in \mathbb{R},\ 0 \le x \le a,$$

where a is a positive constant.

(i) State the largest exact value of a for which the function f^{-1} exists. [1]

For the rest of the question, use the value of a found in part **(i)**.

(ii) Write down the equation of the line in which the graph of $y = \mathrm{f}(x)$ must be reflected in order to obtain the graph of $y = \mathrm{f}^{-1}(x)$ and hence verify that 0 and π are solutions to the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [2]

(iii) Find the area of the region bounded by the graphs of f and f^{-1}, giving your answer in terms of π. [3]

The function g is defined by

$$\mathrm{g} : x \mapsto |x-1|, \quad x \in \mathbb{R}.$$

(iv) Find the exact range of the composite function gf. [2]

2 The angle between two unit vectors **a** and **b** is $\cos^{-1}\frac{1}{4}$. Relative to the origin O, the position vector of a point P on a line l is given by $\overrightarrow{OP} = \mathbf{a} + \lambda(\mathbf{a} + 2\mathbf{b}),\ \lambda \in \mathbb{R}$ and the point C has position vector $\mathbf{a} - \mathbf{b}$.

(i) By considering scalar product, show that $CP^2 = 6\lambda^2 + \frac{9}{2}\lambda + 1$. [4]

(ii) Deduce the exact shortest distance of C to l and write down the position vector of the point F, the foot of the perpendicular from C to l, in terms of **a** and **b**. [3]

(iii) Find the equation of the plane that contains l and is perpendicular to CF in the form $\mathbf{r}.\mathbf{n} = d$ where $\mathbf{n}$ is expressed in terms of **a** and **b** and d is a constant. [2]

3 The number of bacteria (in millions) in Pond A at the start of the nth week, before any chemical treatment, is given by u_n. Pond A is treated at the start of each week with Chemical A, which kills 70% of all bacteria instantly. At the end of each week, 6 million new bacteria is reproduced.

(i) Write down a recurrence relation of the form $u_{n+1} = au_n + b,$ where a and b are constants to be determined. [1]

(ii) Show that $u_n = 0.3^{n-1}u_1 + \frac{60}{7}(1 - 0.3^{n-1}).$ [2]

The number of bacteria (in millions) in Pond B at the start of the nth week, before any chemical treatment, is given by v_n. Pond B is treated at the start of each week with Chemical B. It is known that v_n follows the recurrence relation

$$v_{n+1} = 0.01{v_n}^2 + 6.$$

It is given that if the sequence $v_1, v_2, v_3, \dots$ converges to a limit, it converges to either α or β, where $\alpha < \beta$.

(iii) Find α and β. Explain whether v_n necessarily converges to α or β. [3]

(iv) If $u_1 = v_1 = 30,$ determine which chemical would be more effective in killing the bacteria in the long run. [2]

Pond C is treated with Chemical C. To account for the bacteria's increasing resistance to the chemical, the dosage of Chemical C is increased by 5 ml each week. The first dose is 20 ml.

(v) How many weeks does it take to finish the first 3 litres of chemical in the treatment of Pond C? [3]

4 **Do not use a graphic calculator in answering this question.**

(a) On a single Argand diagram, sketch the locus of z satisfying both inequalities $|z + 1 - 2\mathrm{i}| \le 2$ and $\frac{1}{4}\pi \le \arg(z+1) \le \frac{1}{2}\pi.$ Hence find the range of $\arg(z).$ [5]

(b) Solve the equation

$$w^6 = 64,$$

giving the roots in the form $r\mathrm{e}^{\mathrm{i}\theta},$ where $r > 0$ and $-\pi < \theta \le \pi.$ [2]

(i) Hence write down the roots of the equation $(z - 1 - \mathrm{i}\sqrt{3})^6 = 64$ in the form $a + r\mathrm{e}^{\mathrm{i}\theta},$ where a is a complex number in cartesian form, $r > 0$ and $-\pi < \theta \le \pi.$ Show the roots on an Argand diagram. [3]

(ii) Of the roots found in part **(b)(i)**, find in cartesian form the root with the largest modulus. [2]

Section B: Statistics [60 marks]

5 The Land Transport Authority (LTA) wishes to gather feedback on the quality of train services at a new train station.

(i) The LTA decides to station a team of surveyors at the gantries to survey the first 100 commuters passing through the train station. State, with a reason, whether the method described is quota sampling. [1]

(ii) The LTA decides to obtain a random sample instead to survey 5% of the commuters on a particular day. Describe how a systematic sample can be carried out in this context. [2]

6 John plays for his school's soccer team. There is a probability of 0.15 that he scores in a game and a probability of 0.3 that his parents are present at a game. When he scores in a game, there is a probability of 0.2 that his parents are present.

(i) Show that the probability that he scores in a game when his parents are present is 0.1. [2]

(ii) State, with justification, whether his parents' presence at a game will affect his chances of scoring in the game. [1]

Games are equally likely to be home or away games. In a home game, there is a probability of 0.24 that John does not score and his parents are present.

(iii) Find the least and greatest values of the probability that a game is a home game and his parents are not present at the game. [3]

7 A committee decides to meet on four days in a span of four weeks. Find the probability that the committee meets on two Tuesdays and two Saturdays if

(i) committee meetings are equally likely to be held on any day in the four weeks, [2]

(ii) committee meetings are held once a week. The probability of holding a meeting on any day from Monday to Friday is $\frac{1}{9}$ and the probability of holding a meeting on either Saturday or Sunday is $\frac{2}{9}$. [2]

The committee of ten sits in a circle at a meeting.

(iii) Find the probability that the two committee vice-heads are seated together and they are not seated next to the committee head. [3]

8 A research is being conducted to study the growth of car population over time. The data below shows the population of the car, y millions after x years of study from the start of the research:

Years (x)	5	7	9	14	18	23	27
Car Population (y millions)	7.2	10.5	11.6	13.0	14.5	15.5	15.7

(i) Draw a scatter diagram for the data, labelling the axes. [1]

(ii) State, with a reason, which of the following models is appropriate:

A: $y = a + bx^2$, **B**: $y = a + b\ln x$,

where b is positive. [2]

Based on the appropriate model chosen in part **(ii)**,

(iii) calculate the value of the product moment correlation coefficient. State, with a reason, whether this value would be different if y is recorded in thousands instead. [2]

(iv) calculate the least square estimates of a and b and write down the corresponding regression line. Obtain the value of the car population after 20 years of study. [3]

(v) give an interpretation of the value of a in the context of the question. Comment on the reliability of the value of a. [2]

9 In this question you should state clearly all distributions that you use, together with the values of the appropriate parameters.

(a) The queuing time, in minutes, for flight passengers at the Economy and Business class check-in counters have independent normal distributions with means and standard deviations as shown in the table.

Check-in Counter	Mean queuing time	Standard deviation
Economy class	11.6	4.2
Business class	3.2	0.9

(i) Find the probability that the queueing time of a randomly chosen Economy class passenger is within 5 minutes of the total queueing time of 2 randomly chosen Business class passengers. [4]

(ii) The queueing time of 8 randomly chosen Business class passengers are taken. Find the probability that the shortest queuing time among all 8 passengers is no less than 2 minutes. [2]

(b) The probability that a passenger books a flight and does not turn up is 0.05. The airline decides to allow for over-booking by selling more tickets than the number of seats available.

For a particular flight with 350 available seats, n tickets were sold, where $n > 350$. By using a suitable approximation, show that if the flight is to have no more than 1% chance of having insufficient seats, the number of tickets sold must satisfy the approximate inequality

$$350.5 - 0.95n \geq 2.3263\sqrt{(0.0475n)}.$$ [4]

10 A manufacturer claims that ropes with a certain diameter produced by his factory have mean breaking strength of at least 169.7 kN. Recently, a new material is used to produce the ropes. A random sample of 8 coils of the rope made with the new material is taken and the breaking strength of each coil of rope, x kN, is measured as follows.

171.3	168.5	166.5	164.4	170.0	165.1	170.1	167.2

(i) Find the unbiased estimates of the population mean and variance. [2]

(ii) Stating a necessary assumption, test at the 5% significance level whether the manufacturer's claim is valid after the change in material. [5]

Instead of using the new material, the manufacturer decides to change the weaving process of the ropes. The manufacturer claims that the mean breaking strength is now μ_0 kN. The population variance is found to be $29.16\ (\text{kN})^2$. A random sample of 50 coils of the rope made using the new process is taken and the mean breaking strength, $\bar{y}$ kN, is found to be 171 kN.

(iii) Find the set of values of μ_0 for which the mean breaking strength does not differ from the claim when tested at the 1% significance level. [4]

(iv) Explain, in the context of the question, the meaning of 'at the 1% significance level'. [1]

11 **(a)** A restaurant has 15 tables consisting of 6 rectangular tables and 9 round tables. During the restaurant's opening hours, the rectangular tables are occupied, on average 80 percent of the time, and the round tables are occupied, on average 65 percent of the time. You may assume that the tables in the restaurant are occupied independently of each other.

(i) If a customer walks into the restaurant at a randomly selected time, what is the probability that 4 rectangular tables and 7 round tables are occupied? [2]

(ii) Give a reason in context why the assumption made above may not be valid. [1]

(b) A café sells both coffee and tea. The number of cups of coffee and tea sold in a randomly chosen 20-minute period have independent Poisson distributions with means 5 and 3.5 respectively.

(i) In a particular 20-minute period, at least 7 cups of beverages are sold. Find the probability that at least 6 cups of tea are sold during the 20-minute period. [4]

(ii) Let p_k denote the probability that k cups of coffee are sold in a 20-minute period.

Show that $\dfrac{p_{k+1}}{p_k} = \dfrac{5}{k+1}$ and deduce that $p_{k+1} > p_k$, when $k < 4$. [3]

Hence find the most probable number(s) of cups of coffee sold in a 20-minute period.[2]