Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) 9740/01

Paper 1

14 September 2016

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Score												
Max Score	**6**	**6**	**7**	**7**	**8**	**9**	**10**	**10**	**11**	**13**	**13**	**100**

1 **(i)** Given that $\int \mathrm{f}'(x)\left[\mathrm{f}(x)\right]^n \, \mathrm{d}x = \dfrac{\left[\mathrm{f}(x)\right]^{n+1}}{n+1} + c$ where c is an arbitrary constant and $n \neq -1$, find $\int x\sqrt{(4-x^2)}\, \mathrm{d}x$. [2]

(ii) Hence find the exact volume of revolution when the region bounded by the curve $y = x^{\frac{3}{2}}(4-x^2)^{\frac{1}{4}}$, the lines $x = 0$, $x = 2$ and $y = 3$, is rotated completely about the x-axis. [4]

2 The complex number w is such that $kw^2 + kww^* + \mathrm{i}w - \mathrm{i}w^* - 1 = 0$, where w^* is the complex conjugate of w and k is a real and non-zero constant.

(i) For $w = a + b\mathrm{i}$ where a and b are real numbers, obtain an expression for b in terms of a and k. Explain why w is either purely real or purely imaginary. [4]

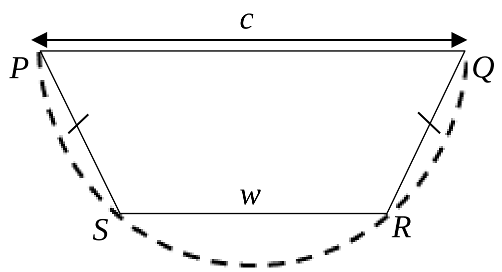
(ii) Using your result in part **(i)**, or otherwise, find the real roots of the equation $2w^2 + 2ww^* + \mathrm{i}w - \mathrm{i}w^* - 1 = 0$. [2]

3 **(i)** Without using a calculator, find the exact solution of the inequality

$$4 - x \geq \frac{4}{x+2}.$$ [4]

(ii) Hence solve $5 - |x| \geq \dfrac{4}{|x|+1}$. [3]

4



To travel along the River Nile, an adventurer decides to use a log with a semi-circular cross-section of constant diameter c metres to build a boat. The log is trimmed such that the uniform cross-section of the boat is an isosceles trapezium with base width w metres and $PS = QR$, as shown in the diagram above.

(i) Show that the cross-sectional area of the boat A metres^2 is given by

$$A = \tfrac{1}{4}(c+w)^{\frac{3}{2}}(c-w)^{\frac{1}{2}}.$$ [2]

(ii) Find the value of w, in terms of c, that gives the stationary value of A. Hence determine whether this stationary value is a maximum or a minimum. [5]

5 Given that $y=\ln(1+\tan x)$,

(i) show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2+2(1-\mathrm{e}^y)\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$, [3]

(ii) find the Maclaurin series for y up to and including the term in x^3, given that the value of $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$ when $x=0$ is 4. [2]

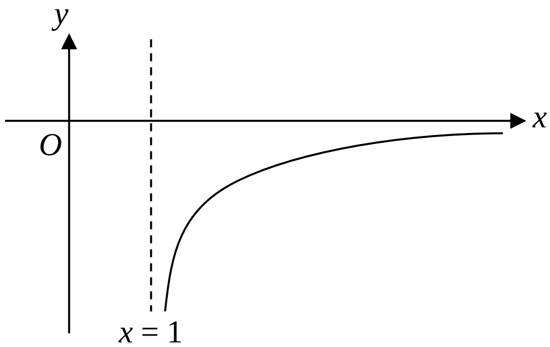
Hence find the first three terms in the series expansion of $\dfrac{\sec^2 x}{1-\tan x}$. [3]

6 **(a)** Use the substitution $x=\mathrm{e}^t$ to find $\displaystyle\int \frac{1}{2\mathrm{e}^t+\mathrm{e}^{-t}}\,\mathrm{d}t$. [3]

(b) **(i)** Express $\dfrac{4+x}{(1-x)(4+x^2)}$ in partial fractions. [2]

(ii) Evaluate $\displaystyle\int_2^n \frac{4+x}{(1-x)(4+x^2)}\,\mathrm{d}x$, giving your answer in the form $\frac{1}{2}\ln\left[\dfrac{\mathrm{f}(n)}{8(n-1)^2}\right]$, where $\mathrm{f}(n)$ is a function of n. [2]

The curve C has equation $y=\dfrac{4+x}{(1-x)(4+x^2)}$. The diagram below shows the part of C for which $x>1$.



Find the exact value of the area of the region between C and the positive x-axis for $x\geq 2$. [2]

7 A curve C has parametric equations

$$x = \frac{\theta}{\sqrt{(1-\theta^2)}}, \quad y = \sin^{-1}\theta, \quad \text{for } -1 < \theta < 1.$$

(i) Show that $\frac{dy}{dx} = 1-\theta^2$. What can be said about the tangents to C as $\theta \to \pm 1$? [4]

(ii) Sketch C, showing clearly its axial intercept and asymptotes. [2]

(iii) Find the equation of the tangent at the point where C has maximum gradient. By considering the intersection between C and an appropriate graph, find the set of positive values of k for which the equation $\sin^{-1}x - \frac{kx}{\sqrt{(1-x^2)}} = 0$ has at most one real root. [4]

8 A sequence of real numbers $u_0, u_1, u_2, \ldots$ satisfy the recurrence relation

$$u_n = u_{n-1} + \ln\left(\frac{n}{n+1}\right)$$

for $n \geq 1$ and $u_0 = 2$.

(i) Use the method of mathematical induction to prove that $u_n = 2 - \ln(n+1)$ for $n \geq 0$. [4]

(ii) By considering $u_r - u_{r-1}$, show how the result for u_n in part **(i)** can be obtained using the method of differences. [4]

(iii) Show that $\sum_{n=0}^{N} u_n > (N+1)(2-\ln(N+1))$. [2]

9 Joseph started a marathon race. After a while, his trainer, Sarah, starts to collect data on Joseph's speed and she realises that the rate of change of Joseph's speed is proportional to the difference between his speed and a constant a. If the speed of Joseph at time t hours after the start of collection of data is u kilometres per hour, it is found that $\frac{\mathrm{d}u}{\mathrm{d}t}=1$ when $u=14.5$ and $\frac{\mathrm{d}u}{\mathrm{d}t}=2$ when $u=14$.

(i) Show that $\frac{\mathrm{d}u}{\mathrm{d}t}=-2(u-15)$. [3]

(ii) Find the general solution of the equation in part **(i)**, expressing u in terms of t. [3]

(iii) Deduce the steady speed of Joseph eventually. [1]

The distance covered by Joseph, s kilometres, at time t hours after the start of collection of data can be modelled by

$$\frac{\mathrm{d}s}{\mathrm{d}t}=u.$$

(iv) Find s in terms of t. [2]

(v) The result in part **(iv)** can be represented by a family of solution curves. Sketch an appropriate non-linear member of the family of curves that has a linear asymptote that passes through the origin. [2]

10 A curve C has equation $y=\frac{x^2-5}{(x+1)^2-12}$.

(i) Determine the equations of the three asymptotes of C, giving each answer in an exact form. [2]

(ii) Prove algebraically that there are no values of x for which $\frac{1}{2}<y<\frac{5}{6}$. [3]

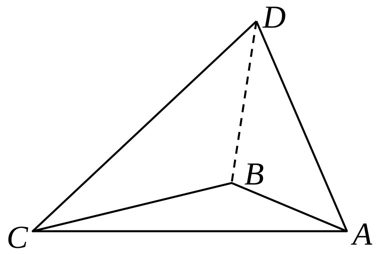
For parts **(iii)** and **(iv)**, you do not need to label the point where the graph cuts the y-axis.

(iii) Sketch C. [3]

(iv) Sketch the graph of $y=\frac{(x+1)^2-12}{x^2-5}$. [3]

(v) Describe a sequence of two transformations which transform C to the graph of $y=\frac{(x-1)^2-5}{(x-2)^2-12}$. [2]

11 The diagram below shows a tetrahedron *ABCD*. The equation of the plane *ABD* is $4x+y+2z=16$.



(i) Given that *A* is on the *x*-axis, find the coordinates of *A*. [1]

The equation of the plane *CBD* is $7x-11y-5z=-23$.

(ii) Find a vector equation of the line that passes through *B* and *D*. [2]

(iii) Given that *B* is on the *xy*-plane, find the coordinates of *B*. [2]

The cartesian equation of the line that passes through *A* and *D* is $\frac{4-x}{2}=\frac{y}{2}=\frac{z}{3}$.

(iv) Find the coordinates of *D*. [3]

The coordinates of *C* are (−1, 1, 1).

(v) By considering the area of triangle *ABC*, find the exact volume of the tetrahedron *ABCD*. [5]

[Volume of tetrahedron = $\frac{1}{3}\times$area of base$\times$perpendicular height]